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An incremental variational formulation for mixed thermo-mechanically coupled problems of nearly incompressible solids

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Extended Abstract

Two variationally consistent methods based on the isogeometric analysis are established to solve the thermo-mechanically coupled problems for a nearly incompressible solid. One is the variationally consistent F-bar projection method, which is the extension of the method proposed by Elguedj et al. [1] for purely mechanical problems. In this method, the standard F-bar projection approach, which introduces two deformation gradients computed at the regular integration point and the centroid, is applied to the variationally consistent thermo-mechanically coupled problems [2].

Another is based on the mixed incremental variational framework of thermo-mechanically coupled problems. In this framework, the instantaneous equilibrium state of the continuum body is identified with the saddle point of the energy rate potential, which consists of the fundamental energy rate potentials and the penalty term to account for incompressibility, and which depends on velocity, external temperature, and pressure variable as global variables, and entropy rate, rate of the internal state variables, and mechanical volume change rate variable as local variables, respectively. To avoid the singularity of the global stiffness matrix and the oscillations of the state variables, we adopt the subdivision stabilization technique using non-uniform rational B-spline (NURBS) function [3]. Specifically, a set of governing equations are spatially discretized by the NURBS basis function, where the NURBS mesh for the extensive variables is twice finer than that for the intensive variables. The suitable combination of the mesh discretization and the polynomial order is studied numerically, allowing robust and stable computations.

Several numerical examples for nearly/perfectly incompressible hyperelastic and standard dissipative solids are presented to investigate the capability of the two methods in terms of computational accuracy, stability, and efficiency. The results demonstrate that the present methods provide the solutions conducive to the quantitative evaluation of the thermo-mechanically coupled behaviors of nearly/perfectly incompressible solids.

References

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